Valid Well-Formed Formulae of Ordinary Propositional Calculus

FR1 A letter standing alone is a wff
FR2 If α is a wff, so is $\sim \alpha$
FR3 If α and β are wff, so is $\alpha \vee \beta$
Def $\wedge$ $(\alpha \wedge \beta) =_{df} \sim (\sim \alpha \vee \sim \beta)$
Def $\supset$ $(\alpha \supset \beta) =_{df} (\sim \alpha \vee \beta)$
Def $\equiv$ $(\alpha \equiv \beta) =_{df} (\alpha \supset \beta) \wedge (\beta \supset \alpha)$
PC1 $(p \wedge q) \supset p$
PC2 $(p \wedge q) \supset q$
PC3 $(p \supset q) \supset ((p \supset r) \supset (p \supset (q \wedge r)))$ (Comp)
PC4 $p \supset (q \supset (p \wedge q))$ (Adj)
PC5 $(p \supset q) \supset ((q \supset p) \supset (p \equiv q))$
PC6 $(p \supset q) \supset ((q \supset r) \supset (p \supset r))$ (Syll)
PC7 $(p \supset (q \supset r)) \supset ((p \wedge q) \supset r)$ (Imp)
PC8 $(p \supset q) \supset ((q \supset (r \supset s)) \supset ((p \wedge r) \supset s))$
PC9 $p \supset (p \vee q)$
PC10 $q \supset (p \vee q)$
PC11 $(p \supset q) \supset ((r \supset q) \supset ((p \vee r) \supset q))$
PC12 $p \equiv \sim\sim p$ (DN)
PC13 $(p \vee q) \equiv \sim (\sim p \wedge \sim q)$ (DeM)
PC14 $(p \wedge q) \equiv \sim (\sim p \vee \sim q)$ (DeM)
PC15 $(p \supset q) \equiv (\sim q \supset \sim p)$ (Transp)
PC16 $(p \vee q) \equiv (q \vee p)$ (Comm)
PC17 $(p \wedge q) \equiv (q \wedge p)$ (Comm)
PC18 $((p \vee q) \vee r) \equiv (p \vee (q \vee r))$ (Assoc)
PC19 $((p \wedge q) \wedge r) \equiv (p \wedge (q \wedge r))$ (Assoc)
PC20 $p \equiv (p \vee p)$
PC21 $p \equiv (p \wedge p)$

Modal Logic

In addition to the rules and formulae of PC:
FR4 If α is a wff, so is $L\alpha$
Def M $M\alpha =_{df} \sim L \sim \alpha$
K $L(p \supset q) \supset (Lp \supset Lq)$ (Neccessarily)
K1 $L(p \wedge q) \supset (Lp \wedge Lq)$
K2 $(Lp \wedge Lq) \supset L(p \wedge q)$
K3 $L(p \wedge q) \equiv (Lp \wedge Lq)$
K4 $Lp \vee Lq) \supset L(p \vee q)$
K5 $Lp \equiv \sim M \sim p$
K6 $M(p \vee q) \equiv (Mp \vee Mq)$
K7 $M(p \supset q) \equiv (Lp \supset Mq)$
K8 $M(p \wedge q) \supset (Mp \wedge Mq)$
K9 $L(p \vee q) \supset (Lp \vee Mq)$
DR1 $\vdash \alpha \supset \beta \rightarrow \vdash L\alpha \supset L\beta$
DR2 $\vdash \alpha \equiv \beta \rightarrow \vdash L\alpha \equiv L\beta$
DR3 $\vdash \alpha \supset \beta \rightarrow \vdash M\alpha \supset M\beta$

D is the extension of K in which every world sees at least one world (serial)

D $Lp \supset Mp$
D1 $M(p \supset p)$
P $\vdash M\alpha \rightarrow \vdash \alpha$

T is the extension of K in which every world sees itself (reflexive)

T $Lp \supset p$
T1 $p \supset Mp$
T2 $M(p \supset Lp)$

S4 is the extension of T (reflexive) with transitivity; S4 has 14 distinct modalities

S4 $Lp \supset LLp$
S4-1 $MMp \supset Mp$
S4-2 $Lp \equiv LLp$
S4-3 $Mp \equiv MMp$
S4-4 $MLMp \supset Mp$
S4-5 $LMp \supset LMLMp$
S4-6 $LMp \equiv LMLMp$
S4-7 $MLP \equiv MLMLp$

B (Boruwerian system) is an extension of T (reflexive) which is symmetric.

B $p \supset LMp$
B1 $MLp \supset p$
DR4 $\vdash M\alpha \supset \beta \rightarrow \vdash \alpha \supset L\beta$

S5 is the extension of T (reflexive) with transivity (S4) and symmetry (B); S5 has 6 distinct modalities

S5 $Mp \supset LMp$
S5-1 $MLp \supset Lp$
S5-2 $Mp \equiv LMp$
S5-3 $Lp \equiv MLp$
S4 $Lp \supset LLp$
S5-4 $L(p \vee Lq) \equiv (Lp \vee Lq)$
S5-5 $L(p \vee Mq) \equiv (Lp \vee Mq)$
S5-6 $M(p \wedge Mq) \equiv (Mp \wedge Mq)$
S5-7 $M(p \wedge Lq) \equiv (Mp \wedge Lq)$